Session 2

Functions, Linear Functions, Graphs, Polynomial Equations, and Percents

Functions

A **function** is a rule or procedure such that when you give it a number, it gives you back one and only one number. For example, the Ideal Body Weight function:

You give it height $\rightarrow$ | IBW function | $\rightarrow$ It gives you IBW

$\uparrow$ This is the "argument" of the function: what you put in

$\downarrow$ $(\text{IBW}) = 106 + 6(h - 60)$

$\uparrow$ This is the "value" of the function: what you get out

Functions

Another function:

You give it gestational age, toxemia	→	Predicting Length of newborn function	→	It gives you length
↑		↓		↑
arguments		length $= 6.61 + 1.05g - 3.48t + 0.06gt$		value

Graphs of Functions

We can call the "value" of a function y and the "argument" of a function x. For every x, we can determine y, and we can plot the points.

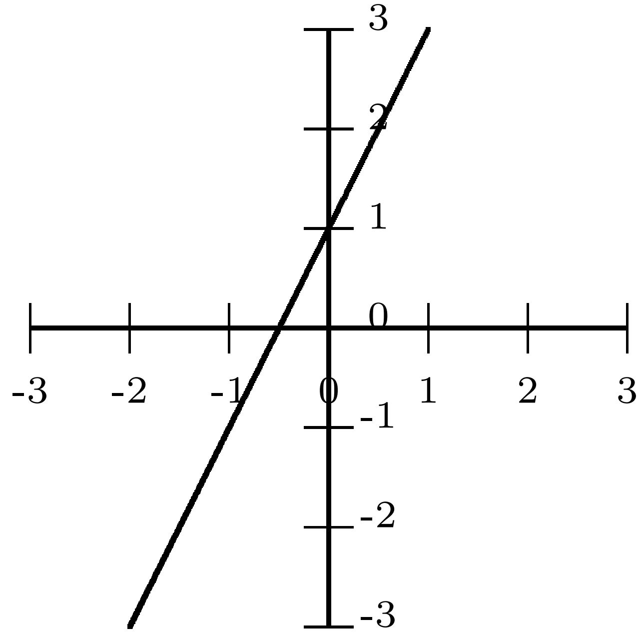
$$y = 2x + 1$$

x	y
-2	-3
-1	-1
0	1
1	3
2	5

Graphs of Functions

$$y = 2x + 1$$

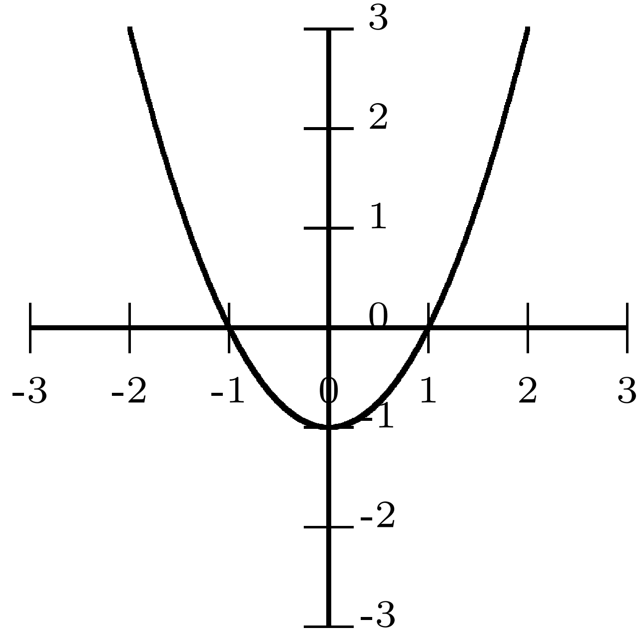
x	y
-2	-3
-1	-1
0	1
1	3
2	5



Graphs of Functions

$$y = x^2 - 1$$

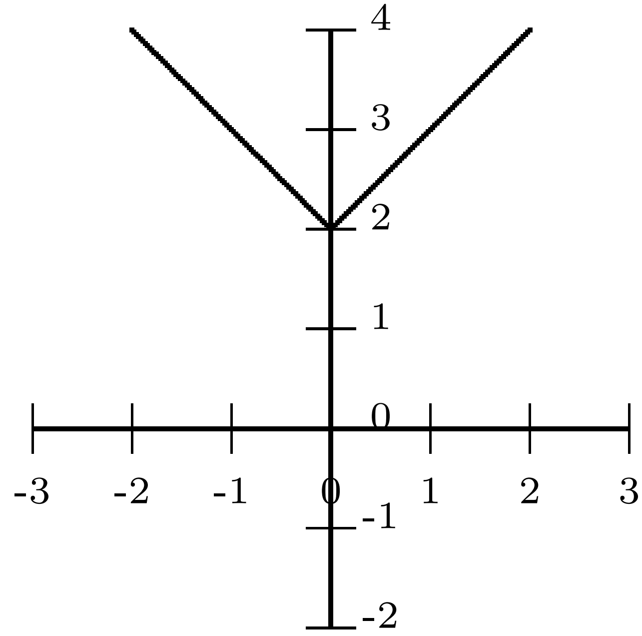
x	y
-2	3
-1	0
0	-1
1	0
2	3



Graphs of Functions

$$y = |x| + 2$$

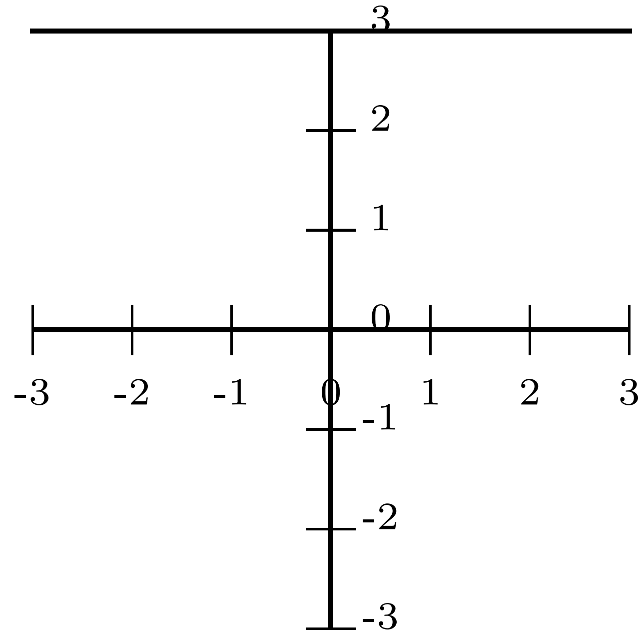
x	y
-2	4
-1	3
0	2
1	3
2	4



Graphs of Functions

$$y = 3$$

x	y
-2	3
-1	3
0	3
1	3
2	3



Linear Functions

A **linear** function is a function whose graph is a straight line. It has the following form (called the slope-intercept form):

$$y = mx + b$$

where m is the **slope** and b is the **y-intercept**, or constant term.

What does the slope mean?

- How steep the line is
- How much y changes with every unit change in x

What does the y-intercept mean?

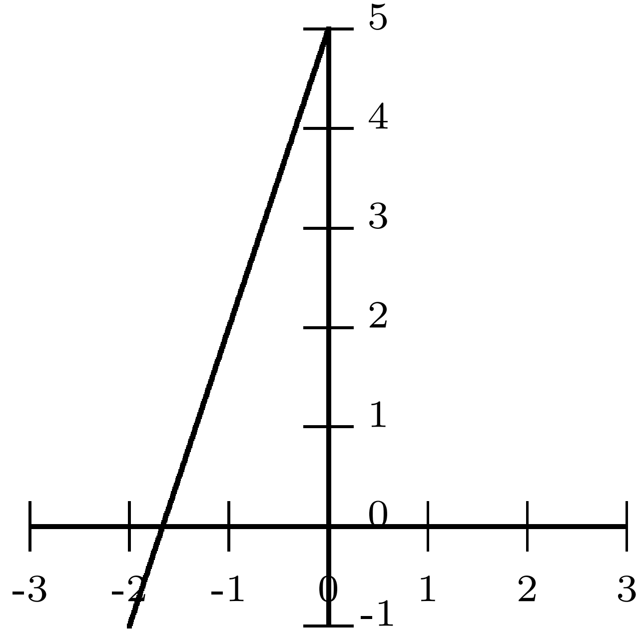
- Where the line crosses the y-axis
- The value of y if $x = 0$

Linear Functions

Examples of linear functions:

$$y = 3x + 5$$

x	y
-2	-1
-1	2
0	5
1	8
2	11

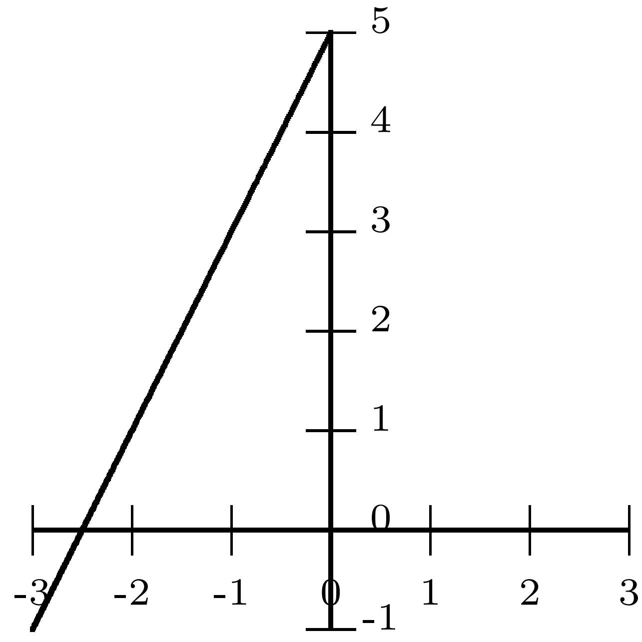


Linear Functions

Examples of linear functions:

$$y = 2x + 5$$

x	y
-2	1
-1	3
0	5
1	7
2	9

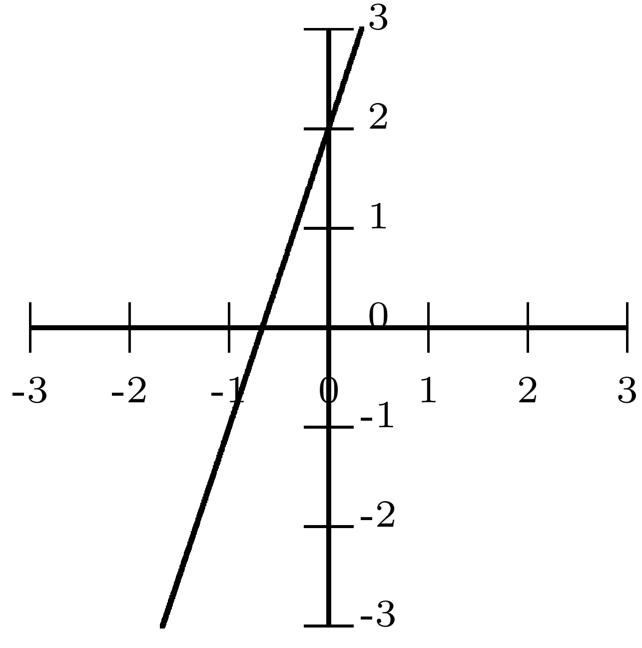


Linear Functions

Examples of linear functions:

$$y = 3x + 2$$

x	y
-2	-4
-1	-1
0	2
1	5
2	8

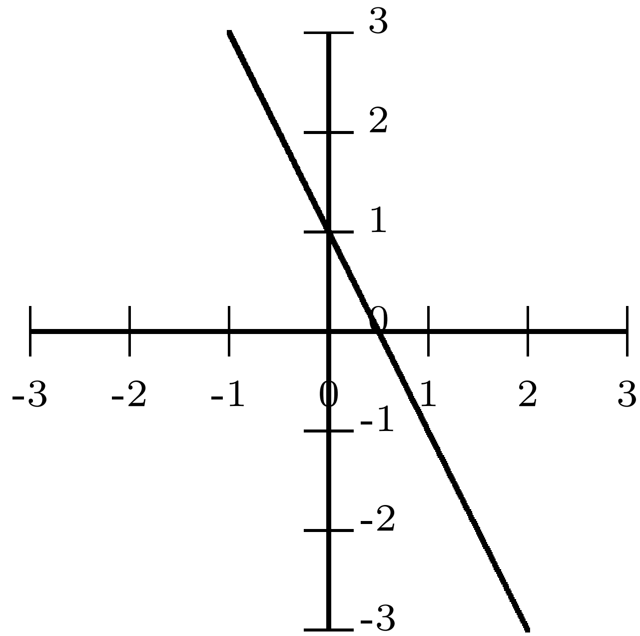


Linear Functions

Examples of linear functions:

$$y = -2x + 1$$

x	y
-2	5
-1	3
0	1
1	-1
2	-3



Linear Functions

Examples of linear functions: Ideal Body Weight for Males

We use y to represent the *value* of the function and x to represent the *argument* of the function. Here, y = ideal body weight and x = height in inches.

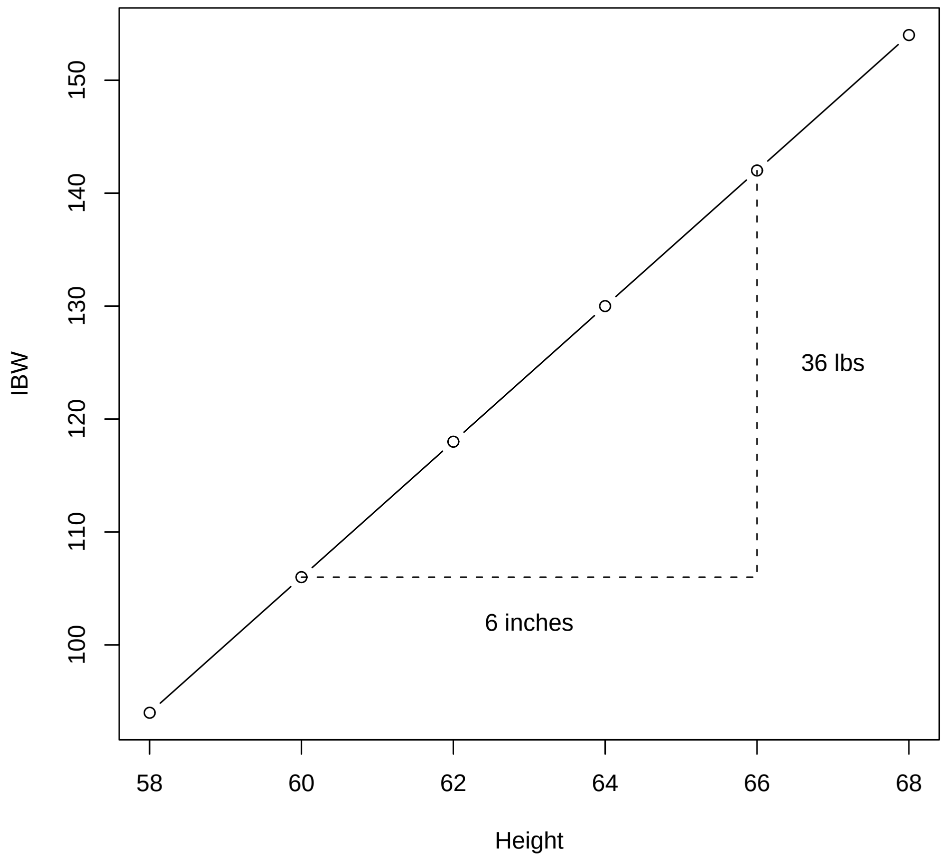
$$y = 106 + 6(x - 60)$$

$$y = 106 + 6x - 360$$

$$y = 6x - 254$$

slope = 6: For every one-inch increase in height, there is a 6 lb increase in ideal body weight.

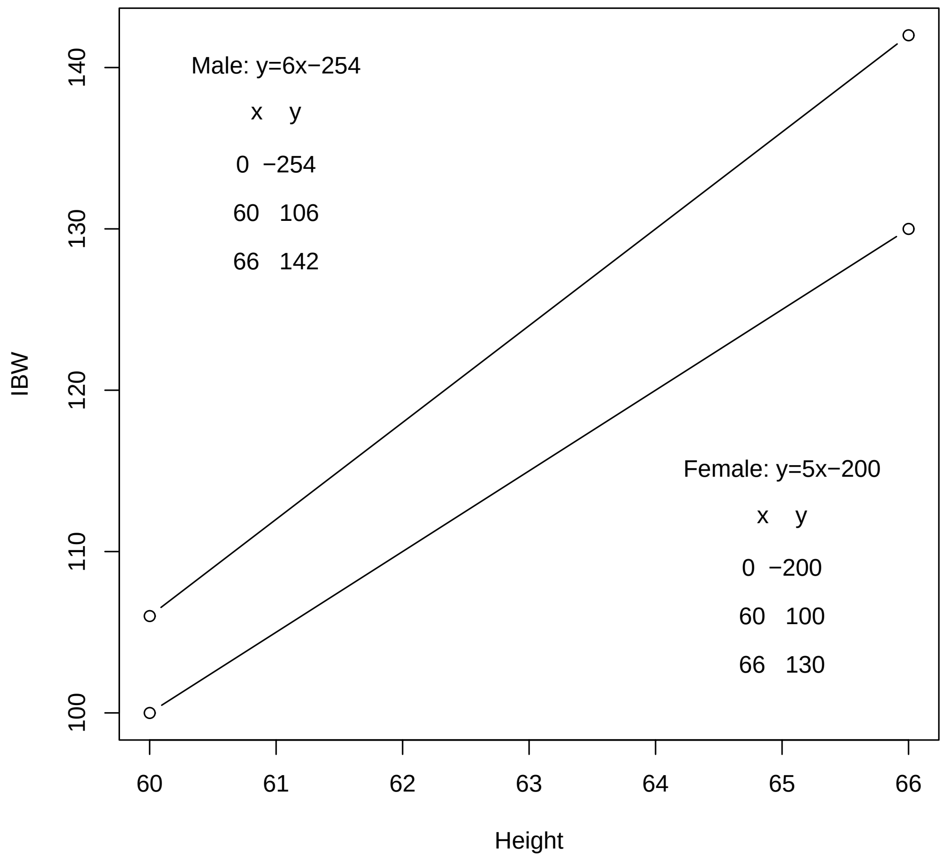
Linear Functions



x	y
0	-254
60	106
66	142

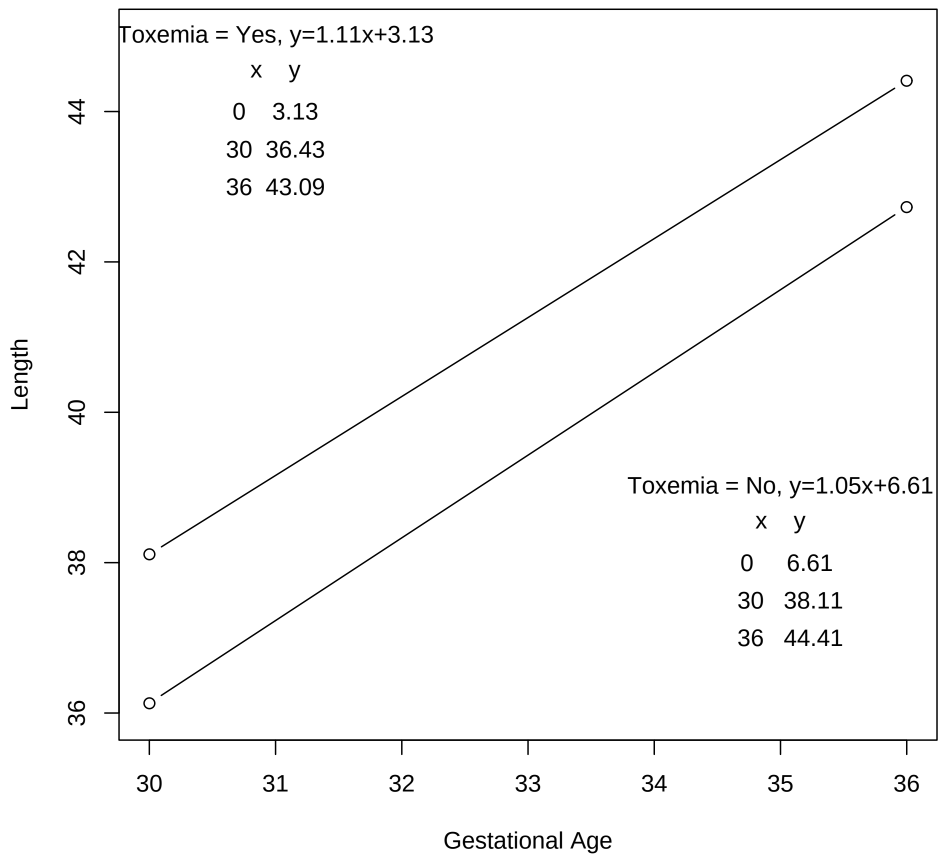
Linear Functions

Compare IBW graphs for males and females:



Linear Functions

Recall our predictions of newborn length from gestational age and toxemia:



Linear Functions

Example (Rosner, p. 483):

A study in El Paso, Texas, looked at the association between lead exposure and neurologic function in children. Neurologic function was measured by the number of finger-wrist taps per 10 seconds in each child's dominant hand. They studied 35 children who had been exposed to lead and 64 children who had not (controls). They found that the following formula described the effects of age, sex, and lead exposure on neurological function:

$$score = 34.1 - 5.1l + 2.4a - 2.4s$$

where l = lead exposure (1=exposed, 0=control), a = age in years, and s = sex (1=male, 2=female).

Linear Functions

We can derive 4 linear equations, one for each combination of sex and lead exposure, by substituting the appropriate values for the two variables.

$$score = 34.1 - 5.1l + 2.4a - 2.4s$$

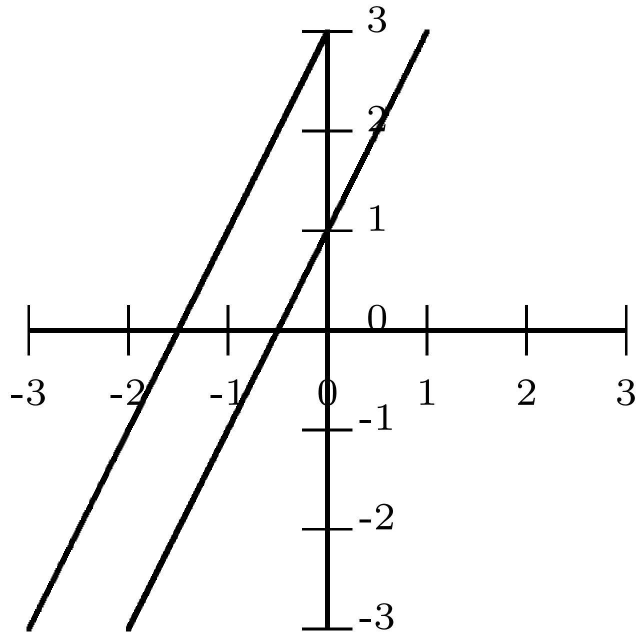
Sex	Lead Exposure	Equation	Simplified Equation
1 (M)	0 (No)	$34.1 + 2.4a - 2.4(1)$	$2.4a + 31.7$
1 (M)	1 (Yes)	$34.1 - 5.1(1) + 2.4a - 2.4(1)$	$2.4a + 26.6$
2 (F)	0 (No)	$34.1 + 2.4a - 2.4(2)$	$2.4a + 29.3$
2 (F)	1 (Yes)	$34.1 - 5.1(1) + 2.4a - 2.4(2)$	$2.4a + 24.2$

Which children scored best? Which children scored worst?

More on Linear Functions

If slopes are the same, then the lines are parallel

$y = 2x + 3 \qquad\qquad y = 2x + 1$

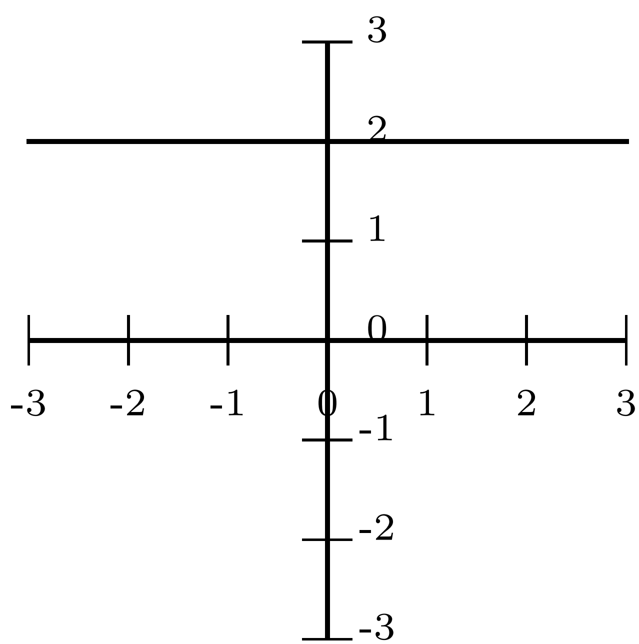


More on Linear Functions

If the slope is 0, the line is horizontal.

$$y = 0x + 2$$

$$y = 2$$

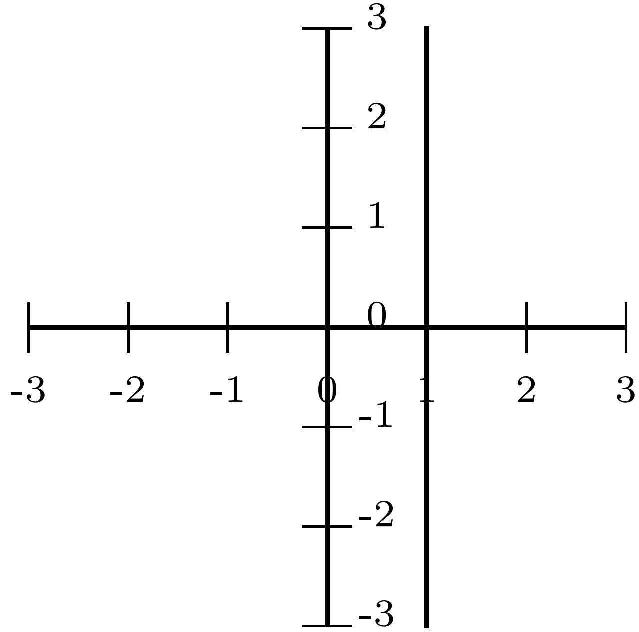


What About a Vertical Line?

A vertical line has infinite slope.

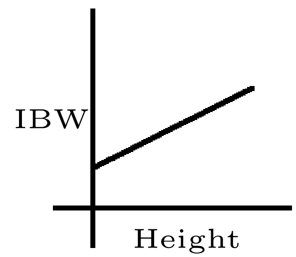
This is not a function!

$$x = 1$$



More on Linear Functions

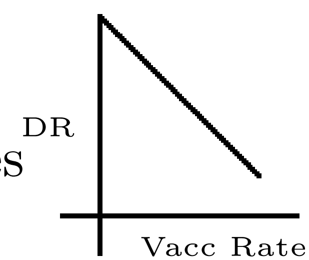
Positive Slope: as x increases, y increases



As height increases, ideal body weight increases

Negative Slope: as x increases, y decreases

As vaccination rate increases, disease rate decreases

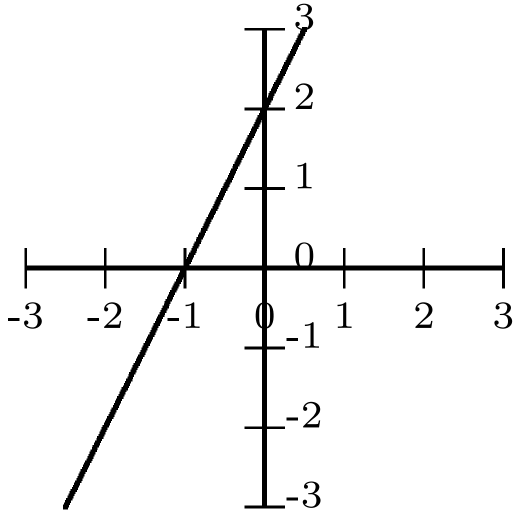


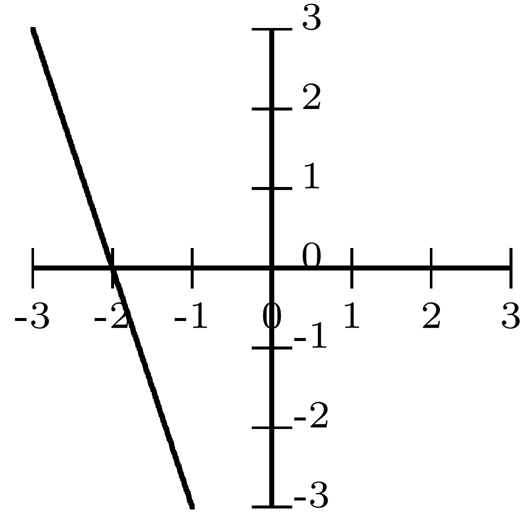
Practice: Linear Functions

Match these equations with their graphs:

$$y = 2x + 2$$

$$y = -3x - 6$$



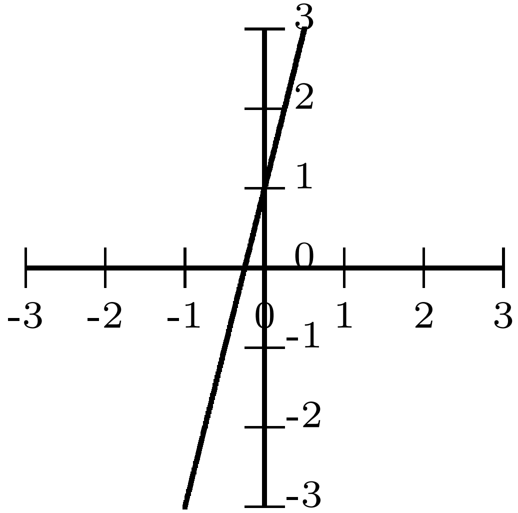


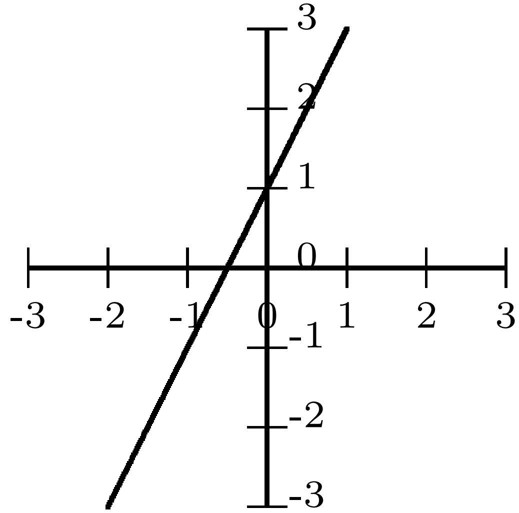
Practice: Linear Functions

Match these equations with their graphs:

$$y = 2x + 1$$

$$y = 1 + 4x$$





Application: Finding Slopes

You have a new drug for prostate cancer that works (you think) by stopping the cancer's growth, but not by killing existing cancer cells. You are following two patients whose only sign of cancer is a rising level of prostate-specific antigen (PSA). You measure the patients' PSA repeatedly, and as soon as it rises above 10 ng/dl, you start them on the experimental drug. Here is what happened to two of your patients. Which one had the best response to the drug?

Patient 1			Patient 2	
Date	PSA (ng/dl)		Date	PSA (ng/dl)
Jan 1	3		Feb 1	6
Aug 1	13	drug started	June 1	20
Dec 1	12		Sept 1	15

Application: Finding Slopes

Recall that we defined slope as how much y changes with a unit change in x. This can be thought of as

$$\frac{\Delta y(\text{change in } y)}{\Delta x(\text{change in } x)}$$

Let's convert the dates in our table to month numbers, and let's consider x to be time and y to be PSA level. We can then compute the slope of the PSA from baseline to treatment, and from treatment to first follow-up.

Patient 1				Patient 2		
Date	Time	PSA		Date	Time	PSA
Jan 1	1	3		Feb 1	2	6
Aug 1	8	13	drug started	June 1	6	20
Dec 1	12	12		Sept 1	9	15

Application: Finding Slopes

Patient 1, Change from baseline to start of treatment

$$\frac{\Delta y}{\Delta x} = \frac{13-3}{8-1} = \frac{10}{7} = 1.43$$

Patient 1, Change from start of treatment to follow-up

$$\frac{\Delta y}{\Delta x} = \frac{12-13}{12-8} = \frac{-1}{4} = -0.25$$

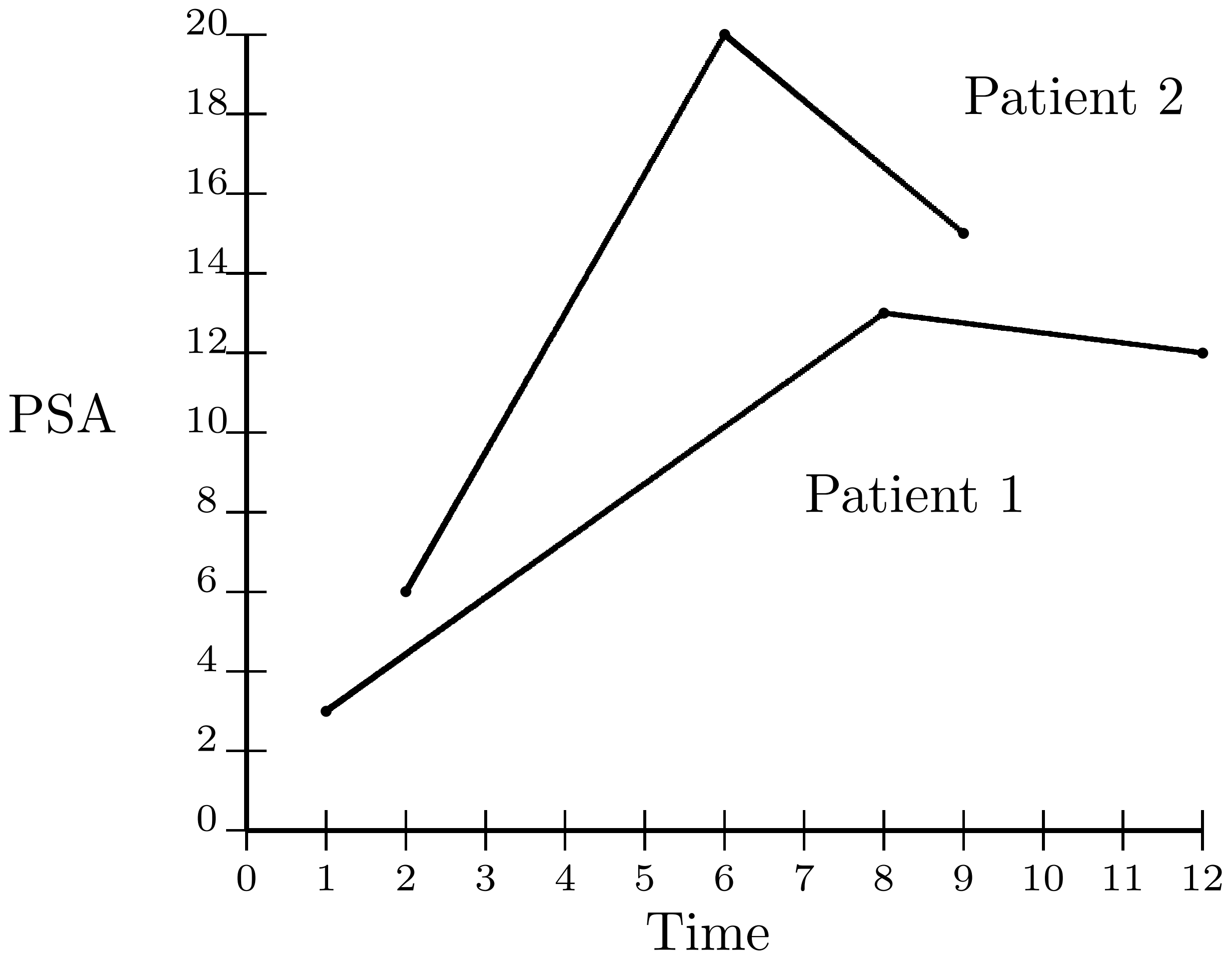
Patient 2, Change from baseline to start of treatment

$$\frac{\Delta y}{\Delta x} = \frac{20-6}{6-2} = \frac{14}{4} = 3.5$$

Patient 2, Change from start of treatment to follow-up

$$\frac{\Delta y}{\Delta x} = \frac{15-20}{9-6} = \frac{-5}{3} = -1.67$$

Application: Finding Slopes



Application: Finding Slopes - Interpreting Results

- Patient 2 appears to have a stronger effect due to treatment, since his slope is steeper than patient 1.
- We need data on more subjects, since we can't tell from just 2 patients whether the difference is due to the treatment, or due to natural variation in PSA behavior.

Polynomial Functions

Linear functions are a special case in a family of functions called **polynomial functions**.

Polynomials are functions with integer powers of x, such as:

$$y = a + bx + cx^2 + dx^3 \dots$$

where a, b, c, d are constants and x is a variable.

The highest power in a polynomial equation is the "degree" of the polynomial.

Linear equations are first degree polynomials.

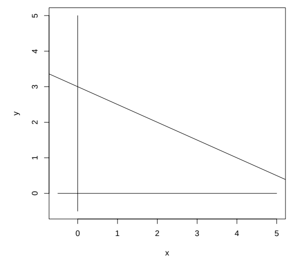
Second degree polynomials are called **quadratics**.

Third degree polynomials are called **cubics**.

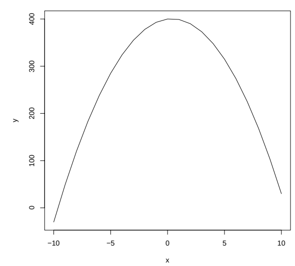
Polynomial Functions

The graph of a polynomial has (degree - 1) turns.

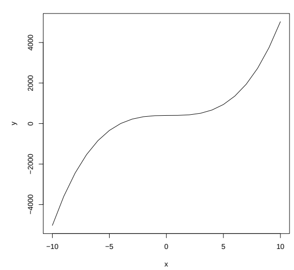
A line has degree 1 and 0 turns



A quadratic has degree 2 and 1 turn



A cubic has degree 3 and 2 turns



Polynomial Functions: Example

A study was done to determine the dose of a drug that would best promote weight gain in laboratory animals [Kleinbaum, Kupper and Muller, page 233]. 8 animals, all of the same sex, age, and size, were randomly assigned to 1 of 8 dose levels. They were cared for in identical conditions for 2 weeks, and were then weighed. Weight gain was defined as the difference between ending weight and starting weight, measured in dekagrams. 1 dekagram = 10 grams = .35274 ounce.

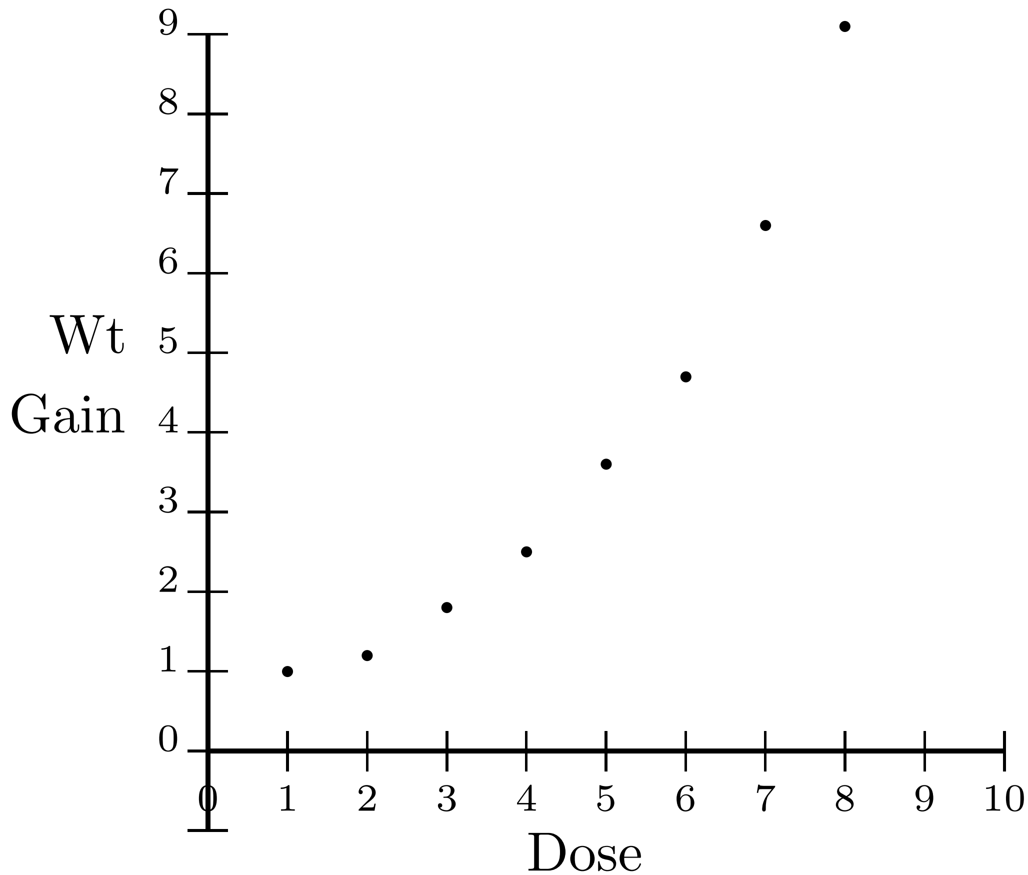
Polynomial Functions: Example

The weight gain for each dose level in dekagrams is shown below:

Dose	Weight Gain (dekagrams)
1	1
2	1.2
3	1.8
4	2.5
5	3.6
6	4.7
7	6.6
8	9.1

Polynomial Functions: Example

Here is a graph of the points:



Polynomial Functions: Example

Do the points fall in a straight line?

No! We may need a quadratic equation to describe the relationship between weight gain and dose.

Polynomial regression was used to find the equation with the best fit. Here is that polynomial (quadratic) function:

$$y = 1.13 - 0.41x + 0.17x^2$$

where $x =$ dose and $y =$ weight gain.

Everything does not have to follow a linear function!

Percents

Percent is defined as a rate or proportion per hundred, 1/100.

Example: $1\% = \frac{1}{100} = 0.01$

1% of $9 = 0.01 \times 9 = .09$

You can add and subtract percents straightforwardly:

$$25\% + 5\% = 30\%$$

To multiply or divide percents, convert them to their decimal equivalents first:

$$25\% \times 5\% = 0.25 \times 0.05 = 0.0125$$

To convert the number back to a percent, multiply by 100:

$$0.0125 = 1.25\%$$

Percent Increase or Decrease

To determine the percent that a number has increased or decreased, use this equation:

$$\text{percent change} = \frac{\text{final - beginning}}{\text{beginning}} \times 100$$

Example: If 30% of patients lose weight with conventional programs and your new program shows that 40% of patients lose weight, what percent improvement are you demonstrating?

$$\begin{aligned} \text{percent change} &= \frac{.40 - .30}{.30} \times 100 = \frac{.10}{.30} \times 100 \\ &= \frac{1}{3} \times 100 = .333 \times 100 = 33.3\% \end{aligned}$$

Practice with Percents

25% of 60 =

$\frac{3}{5}$ =?%

An increase from 10 to 12 is what percent increase?

$10\% \times 5\% =$

Convert 75% to a fraction:

Convert 35% to a decimal:

Convert 0.075 to a percent:

Solutions to Practice Problems

Slide 23

Left: $y = 2x + 2$

Right: $y = -3x - 6$

Slide 24

Left: $y = 1 + 4x$

Right: $y = 2x + 1$

Solutions to Practice Problems

Slide 38:

25% of 60 = 15

$\frac{3}{5} = 60\%$

An increase from 10 to 12 is a 20% increase

$10\% \times 5\% = 0.5\%$

Convert 75% to a fraction: $\frac{3}{4}$

Convert 35% to a decimal: 0.35

Convert 0.075 to a percent: 7.5%