Doing Math: Scientific Notation

Scientific notation is a way to show (and do math with) very small or very large numbers

$$b \times 10^c$$

$b =$ a decimal number between 1.0 and $9.99\ldots$

$c =$ an exponent of 10

Example:

$$1.53 \times 10^3 = 1530$$

$$2.4 \times 10^{-2} = 0.024$$

To convert from scientific notation to standard notation, move the decimal c places to the right. (This is c places to the right if $c > 0$ and $|c|$ places to the left if $c < 0$).

Doing Math: Scientific Notation

$$b \times 10^c$$

Multiplying 2 numbers in scientific notation:

- Multiply the b's
- Add the c's
- Round to the same or fewer decimal places than the least precise factor

Example:

$$(1.76 \times 10^{-8}) \times (3.2 \times 10^{12}) = (1.76 \times 3.2) \times 10^{(12-8)}$$

$$= 5.632 \times 10^4$$

3.2 is least precise factor, so round to 5.6×10^4

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$$b \times 10^c$$

Dividing 2 Numbers in Scientific Notation:

- Divide the b's
- Subtract the c's
- Round to the same or fewer decimal places than the least precise factor

Example:

$$(3.000 \times 10^9)/(2.1065 \times 10^{13}) = \frac{3.000}{2.1065} \times 10^{(9-13)}$$

$$= 1.4241... \times 10^{-4}$$

3.000 is least precise factor, so round to 1.424×10^{-4}

Doing Math: Scientific Notation

$$b \times 10^c$$

Adding and Subtracting in Scientific Notation:

If the c's are the same:

- Add or subtract the b's
- Keep the same c
- Round to the same or fewer decimal places than the least precise factor
- Rescale if b is less than 1.0 or greater than or equal to 10.0

What if the c's are not the same?

- Realign to get the same c's, then add or subtract
- Use a calculator

Doing Math: Scientific Notation

$$b \times 10^c$$

Examples:

$$1.53 \times 10^3 + 2.42 \times 10^3 = (1.53 + 2.42) \times 10^3 = 3.95 \times 10^3$$

$$2.4 \times 10^2 + 2.82 \times 10^2 = 5.22 \times 10^2$$

Round to number of digits in least precise number $\rightarrow= 5.2 \times 10^2$

$$4.78 \times 10^5 + 8.4 \times 10^5 = 13.2 \times 10^5$$

Keep b between 1.0 and $9.99\ldots$, so $\rightarrow= 1.32 \times 10^6$

Example: Calculating the Standard Deviation of n BMI's

Standard deviation: a measure of how widely scattered values are (More on this in your first stats class...).

n = how many

To compute:

1. Compute the average
2. Subtract the average from each BMI
3. Square the differences
4. Sum the squared differences
5. Divide by n-1
6. Compute the square root

Example: Standard Deviation

BMI		Avg		Diff	Sq Diff
22.2	-	26.76	=	-4.56	20.79
23.1	-	26.76	=	-3.66	13.40
35.0	-	26.76	=	8.24	67.90
28.3	-	26.76	=	1.54	2.37
19.5	-	26.76	=	-7.26	52.71
22.5	-	26.76	=	-4.26	18.15
37.4	-	26.76	=	10.64	113.21
30.0	-	26.76	=	3.24	10.50
29.1	-	26.76	=	2.34	5.48
20.5	-	26.76	=	-6.26	39.19

Example: Standard Deviation

Sum of squared differences = 343.70

Divide by n-1 = 10-1 = 9 to get 38.18888

Take the square root = 6.18, or 6.2

Later we will use a statistical software package to calculate means (averages) and standard deviations.

Algebraic Expressions: Order of Operations

Order is important!

Recall $BMI = \frac{\text{Wt}}{\text{Ht}^2}$

For example, $BMI = \frac{67.5}{1.7^2} = 23.4$

We squared Ht *before* dividing Wt by the denominator.

What if we divided and *then* squared?

$$BMI \neq \left(\frac{\text{Wt}}{\text{Ht}}\right)^2 = \left(\frac{67.5}{1.7}\right)^2 = 1576.3$$

Not the same!

Algebraic Expressions: Order of Operations

Solution: "Paul and Judi's Rules of Order"

1. Do operations in **P**arentheses first, starting with the innermost brackets.
2. Simplify **E**xponential expressions.
3. **M**ultiply and **D**ivide from left to right.
4. **A**dd and **S**ubtract from left to right.

"**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally"

Algebraic Expressions: Order of Operations

Example:

$$\left[5+\frac{(3+1)^2}{2}\times 2-\frac{2}{1}+1\right]^2$$

1. Innermost bracket $\left[5+\frac{4^2}{2}\times 2-\frac{2}{1}+1\right]^2$
2. Exponents inside bracket $\left[5+\frac{16}{2}\times 2-\frac{2}{1}+1\right]^2$
3. Multiplications and divisions inside bracket $[5+8\times 2-2+1]^2$

$$[5+16-2+1]^2$$

4. Additions and subtractions inside bracket, left to right $[20]^2$
5. Exponents outside bracket $= 400$

Practice: Order of Operations

$$\frac{\left[22+(7-3)^2+\frac{20}{4}\times 3+\frac{27-2+3}{2}\right]^2}{2}$$

$$\frac{\left[22+(4)^2+\frac{20}{4}\times 3+\frac{28}{2}\right]^2}{2}$$

$$\frac{\left[22+16+\frac{20}{4}\times 3+\frac{28}{2}\right]^2}{2}$$

$$\frac{[22+16+5\times 3+14]^2}{2}$$

Practice: Order of Operations

$$\frac{[22 + 16 + 15 + 14]^2}{2}$$

$$\frac{[67]^2}{2}$$

$$\frac{4489}{2}$$

$$2244.5$$

Order of Operations

You can use brackets and parentheses to add clarity to formulas, even if the formula is “correct” without them. They can help you remember which operations to do first.

Be liberal with brackets and parentheses!

Order of Operations: Practice with a Calculator

$$\frac{\left[22 + (7-3)^2 + \frac{20}{4} \times 3 + \frac{27-2+3}{2}\right]^2}{2}$$

$$((22 + (7-3)^2 + (20/4) * 3 + ((27 - 2 + 3)/2))^2)/2$$

Variable Expressions

When we use letters as placeholders for unknown quantities, the letters are called “variables”.

We saw $b \times 10^c$ when we discussed scientific notation.

We could write BMI $= \frac{w}{h^2}$

Sometimes we write xy to mean “x times y”.

If $x = 3$ and $y = 2$ then $xy = 6$ (not 32)

Evaluating Variable Expressions

Substitute for each variable the number it represents.

$$\text{BMI} = \frac{w}{h^2}$$

Let $w = 67.5$ kg and $h = 1.7$ m.

$$\text{BMI} = \frac{67.5\ \text{kg}}{1.7^2\ \text{m}^2} = 23.4$$

Evaluating Variable Expressions

Ideal body weight (IBW) for males, where $h =$ height in inches:

$$\begin{aligned}\text{IBW (lbs)} &= 106 + 6(h - 60) \\ &= 106 + (6 * (h - 60))\end{aligned}$$

Let $h = 68$ inches.

$$\begin{aligned}\text{IBW (lbs)} &= 106 + (6 * (68 - 60)) \\ &= 106 + (6 * 8) \\ &= 106 + 48 \\ &= 154 \text{ lbs}\end{aligned}$$

Simplifying Variable Expressions

If two terms have the same variables, they can be combined.

These can be combined:

$$\begin{aligned} 3x^2y + 2x^2y &= 5x^2y \\ 2a^2b + 2ba^2 &= 4a^2b \qquad \text{(commutative law)} \end{aligned}$$

These cannot be combined:

$$3x^2y + 2x^2$$

$$2a^2b + 2ab^2$$

Practice Simplifying Variable Expressions

$$3a + 4b + 7 - 2a + 8 + 6b^2 + 3b$$

$$= a + 7b + 15 + 6b^2$$

Evaluating Variable Expressions

We can use *regression* to predict the length of a newborn baby, given two factors we know about the baby's birth:

- Did the mother experience toxemia during pregnancy?
- What was the baby's gestational age?

Let l = length in centimeters

Let g = gestational age in weeks

Let t = toxemia (where 0 = no toxemia and 1 = toxemia)

$$l = 6.61 + 1.05g - 3.48t + 0.06gt$$

Evaluating Variable Expressions

What if toxemia = yes? Then $t = 1$ and:

$$l = 6.61 + 1.05g - 3.48 + 0.06g$$

Simplifying:

$$l = (6.61 - 3.48) + (1.05 + 0.06)g$$

$$l = 3.13 + 1.11g$$

Evaluating Variable Expressions

What if toxemia = no? Then $t = 0$ and:

$$l = 6.61 + 1.05g - 3.48(0) + 0.06(0)g$$

$$l = 6.61 + 1.05g$$

So, for toxemia = 1 (yes), $l = 3.13 + 1.11g$.

For toxemia = 0 (no), $l = 6.61 + 1.05g$.

Evaluating Variable Expressions

$$l = 6.61 + 1.05g - 3.48t + 0.06gt$$

If gestational age = 32 weeks and toxemia = yes, then what is the infant's predicted length? We can substitute:

$$l = 6.61 + 1.05(32) - 3.48(1) + 0.06(32)(1)$$

Simplifying (first multiply, then add and subtract):

$$l = 6.61 + 33.6 - 3.48 + 1.92$$

$$l = 38.65 \text{ cm}$$