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Math Workshop On-Line Tutorial

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Session 1

Kinds of Numbers and Data, Fractions,

Negative Numbers, Rounding, Averaging,

Properties of Real Numbers,

Exponents and Square Roots,

Scientific Notation, Order of Operations,

Evaluating Variable Expressions

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Kinds of Numbers and Data

First we're going to think about the kinds of numbers you will use in the problems you will encounter in your studies. Then we will expand a bit and think about kinds of data. Some data elements are numbers, but some are not. There are a variety of ways to think about these data elements.

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An Example

Suppose you have conducted a group weight loss program and are interested in determining how effective it was. You are interested in investigating whether this approach to weight loss works equally well in members of all racial/ethnic groups, and in both men and women. You might first want to know about the program's participants. Here is what you learn about their gender, race, number of previous attempts to lose weight, Body Mass Index (BMI), starting weight in pounds, and each participant's rank when ordered by starting weight:

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Efficacy of a Weight Loss Program

Sex	Race	Previous Attempts	BMI Category	Starting Weight (pounds)	Rank
Female	Caucasian	2	Normal	135.25	2
Female	Black	4	Overweight	159.82	3
Male	Black	3	Normal	115.343	1
Male	Hispanic	2	Obese	190.212	4
Female	Caucasian	3	Sev. Obese	274.931	5

1 pound (lb) = 0.45 kg, BMI = Body Mass Index

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Efficacy of a Weight Loss Program

On the following slide you will find once again the sex (gender) and race of each participant and the starting weight in pounds. This time, the weight has been rounded to a whole number (more on that later). You will also find the ending weight and the amount of weight lost by each participant. Weight loss was determined by subtracting the ending weight from the starting weight. Sometimes this is a positive number (the participant lost weight) and sometimes it is a negative number (the participant gained weight).

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Efficacy of a Weight Loss Program

Sex	Race	Starting Weight (lbs)	Ending Weight (lbs)	Weight Loss (lbs)
Female	Caucasian	135	140	-5
Female	Black	160	145	15
Male	Black	115	130	-15
Male	Hispanic	190	140	50
Female	Caucasian	275	300	-25

1 lb = 0.45 kg

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Kinds of *Numbers*

- **Integers or Whole Numbers**

  "How many participants lost weight?"

  - Can be Positive
  - Can be Negative
  - Can be Zero (neither positive nor negative)
  - Examples:
    - Number of live births a woman has had
    - Number of previous attempts to quit smoking
    - Number of points systolic blood pressure decreased after an intervention

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Kinds of *Numbers*

- **Fractions, Decimals and Percents**

  "What proportion of participants lost weight?"
  - Fractions: Proportions expressed as ratios: 1/3
  - Decimals: Proportions xpressed with decimal points: 0.333
  - Percents: Decimals expressed in terms of 100: 33.3% = 0.333
  - Examples:
    - Fraction of women in a prenatal program who have had 2 previous live births
    - Proportion of participants in program to quit smoking who have had no prior attempts
    - Percent reduction in systolic blood pressure

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Kinds of *Data*

- **Nominal Data**
  - "Named"
  - Categories
  - No particular order
  - Special Case: **Dichotomous data** - Nominal data with 2 categories
  - Examples:
    - Sex (Nominal and Dichotomous)
    - Race (Nominal)
    - Eye Color (Nominal)
    - Lab Test Result - Positive or Negative (Nominal and Dichotomous)

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Kinds of Data

- **Ordinal Data**
  - "Ordered"
  - Categorical
  - Categories have order
  - No guarantee that magnitude is the same across categories
  - Examples:
    - Body Mass Index category (low, medium, high)
    - Letter grades A, B, C, D, F
    - Ratings of excellent, good, fair, poor

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Kinds of Data

- **Ranked Data**
  - Values sorted from highest to lowest and then assigned a number representing the position, or rank
  - Examples:
    - Rank by starting weights of program participants
    - Rank by dollars of revenue of commercial weight loss programs in the United States
    - Ranks of causes of death in a country or region

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Kinds of Data

- **Discrete Data**
  - Both order and magnitude are important
  - Restricted to taking on specific values
  - Can use arithmetic to summarize
  - Frequently, these are "count" data
  - Examples:
    - Number of previous weight loss attempts by program participants
    - Number of traffic accidents at Brigham Circle in one week

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Kinds of Data

- **Continuous Data**
  - Any value, integer or fraction, is possible
  - Frequently the result of taking a measurement: time, temperature, cholesterol, weight
  - Accuracy depends on precision of measurement
  - Can use math to summarize by averaging, counting the number above and below average
  - Examples:
    - Starting weight of program participants
    - Height of program participants

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Practice (Solutions are at the End)

Based on the weight loss program data, identify the kind of number and/or data and answer these questions:

How many Hispanic males?

Who weighed the most at program entry?

How many participants weighed more than 150 pounds at program entry?

What kind of number and what kind of data is *weight loss*?

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Doing Math: Fractions

We had 2 men in a class of 5 participants.

$$\frac{numerator \rightarrow 2}{denominator \rightarrow 5} \leftarrow \text{“}divide\ by\text{”} = 0.4$$

What if we had a class of 50 with 20 men?

If the numerator and denominator can be divided by the same number, you can "reduce" the fraction.

$$\underbrace{\frac{20}{50} = \frac{2 \times 10}{5 \times 10} = \frac{2}{5}}_{equivalent}$$

What is the fraction of non-Caucasian participants?

$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

The denominators are the same, so we can add.

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Doing Math: Fractions

What if the denominators are not the same?

Class A: 5 participants: 2 Black, 1 Hispanic, 2 Caucasian

Class B: 6 participants: 2 Black, 2 Hispanic, 2 Caucasian

We want to know the "average" number of non-Caucasian participants. We'll add the two classes, then divide by 2.

Class A:

$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

Class B:

$$\frac{2}{6} + \frac{2}{6} = \frac{4}{6} = \frac{2 \times 2}{3 \times 2} = \frac{2}{3}$$

First, we add:

$$\frac{3}{5} + \frac{2}{3} = \frac{3 \times 3}{5 \times 3} + \frac{2 \times 5}{3 \times 5} = \frac{9}{15} + \frac{10}{15} = \frac{19}{15}$$

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Doing Math: Fractions

Now we divide $\frac{19}{15}$ by 2:

$$2 = \frac{2}{1}$$

$$\frac{19}{15} \div \frac{2}{1} = \underbrace{\frac{19}{15} \times \frac{1}{2}}_{\textit{invert and multiply}} = \frac{19 \times 1}{15 \times 2} = \frac{19}{30}$$

In the above calculation, we gave each *class* equal weight when we took the total and divided by 2. However, the classes are not the same size. What if we gave each *participant* equal weight? (This is a more standard approach.)

$$\frac{4\ \textit{Black participants}}{11\ \textit{total participants}} + \frac{3\ \textit{Hispanic participants}}{11\ \textit{total participants}} = \frac{7}{11}$$

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Doing Math: Fractions

Which is bigger? Fractions can be hard to compare, so we'll convert the fractions to decimals:

$$\frac{19}{30} = 0.6333 \qquad \frac{7}{11} = 0.6363$$

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Practice Doing Math with Fractions

$$\frac{1}{7} + \frac{3}{7} \qquad \frac{3}{8} - \frac{1}{8}$$

$$\frac{3}{7} + \frac{1}{8} \qquad \frac{3}{5} + \frac{5}{2}$$

$$\frac{3}{8} \times \frac{1}{8} \qquad \frac{3}{8} * \frac{1}{4}$$

$$\frac{3}{8} \div \frac{1}{4}$$

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Doing Math with Fractions on a Calculator

The calculator multiplies and divides from left to right, then adds and subtracts from left to right. It will do operations in parentheses first, though, so you can force it to change its order.

Addition:

$\frac{2}{3} + \frac{3}{5} =$ [2] [÷] [3] [+] [3] [÷] [5] [=] 1.267

Subtraction:

$\frac{2}{3} - \frac{3}{5} =$ [2] [÷] [3] [−] [3] [÷] [5] [=] 0.067

Multiplication:

$\frac{2}{3} - \frac{3}{5} =$ [2] [÷] [3] [×] [3] [÷] [5] [=] 0.4

Division:

$\frac{2}{3} \div \frac{3}{5} =$ [2] [÷] [3] [÷] [3] [÷] [5] [=] 0.067? WRONG!

$\frac{2}{3} \div \frac{3}{5} =$ [(] [2] [÷] [3] [)] [÷] [(] [3] [÷] [5] [)] [=] 1.111

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Practice Doing Math with Fractions: Calculator

$\frac{1}{7} + \frac{3}{7}$

$\frac{3}{8} - \frac{1}{8}$

$\frac{3}{7} + \frac{1}{8}$

$\frac{3}{5} + \frac{5}{2}$

$\frac{3}{8} \times \frac{1}{8}$

$\frac{3}{8} * \frac{1}{4}$

$\frac{3}{8} \div \frac{1}{4}$

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Doing Math: Rounding

We can *round* numbers to make them easier to understand and compare. It is important not to imply that numbers have great precision, when we simply have failed to round off. It's best to work with extra decimal places while doing calculations, then round off the answer as the last step in solving a problem.

$\frac{19}{30} = 0.6333$ $\qquad$ $\frac{7}{11} = 0.6363$

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Doing Math: Rounding

Know Your Digits:

1 0 0 0.0 0 0 0

Thousand
Hundred
Ten
One
Tenth
Hundredth
Thousandth
Ten Thousandth

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Doing Math: How to Round

1. Look at the digit to the right of the one you want to keep.
2. Is it 5 or greater? Then increase the digit you want to keep by 1.
3. Is it 4 or smaller? Then leave the digit you want to keep alone.
4. If the "keeper" digit is to the right of the decimal, drop all digits farther to the right.
5. If the "keeper" digit is to the left of the decimal, change all digits farther to the right to 0.

Example: Round to the nearest hundredth:

$0.6363 \rightarrow 0.64$ $0.6333 \rightarrow 0.63$

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Practice Rounding:

855.23 round to tens

round to tenths

round to ones
(nearest integer)

round to hundreds